XV. *Account of the Changes that have happened, during the last Twenty-five Years, in the relative Situation of Double-stars; with an Investigation of the Cause to which they are owing. By* William Herschel, *LL. D. F. R. S.*

Read June 9, 1803.

In the Remarks on the Construction of the Heavens, contained in my last Paper on this subject,* I have divided the various objects which astronomy has hitherto brought to our view, into twelve classes. The first comprehends insulated stars.

As the solar system presents us with all the particulars that may be known, respecting the arrangement of the various subordinate celestial bodies that are under the influence of stars which I have called insulated, such as planets and satellites, asteroids and comets, I shall here say but little on that subject. It will, however, not be amiss to remark, that the late addition of two new celestial bodies, has undoubtedly enlarged our knowledge of the construction of the system of insulated stars. Whatever may be the nature of these two new bodies, we know that they move in regular elliptical orbits round the sun. It is not in the least material whether we call them asteroids, as I have proposed; or planetoids, as an eminent astronomer, in a letter to me, suggested; or whether we admit them at once into the class of our old seven large planets. In the latter case, however, we must recollect, that if we would speak with precision,

* See Phil. Trans. for 1802, p. 477.

they should be called very small, and exzodiacal; for, the great inclination of the orbit of one of them to the ecliptic, amounting to 35 degrees, is certainly remarkable. That of the other is also considerable; its latitude, the last time I saw it, being more than 15 degrees north. These circumstances, added to their smallness, show that there exists a greater variety of arrangement and size among the bodies which our sun holds in subordination, than we had formerly been acquainted with, and extend our knowledge of the construction of the solar, or insulated sidereal system. It will not be required that I should add any thing farther on the subject of this first article of my classification; I may therefore immediately go to the second, which treats of binary sidereal systems, or real double stars.

We have already shewn the possibility that two stars, whatsoever be their relative magnitudes, may revolve, either in circles or ellipses, round their common centre of gravity; and that, among the multitude of the stars of the heavens, there should be many sufficiently near each other to occasion this mutual revolution, must also appear highly probable. But neither of these considerations can be admitted in proof of the actual existence of such binary combinations. I shall therefore now proceed to give an account of a series of observations on double stars, comprehending a period of about 25 years, which, if I am not mistaken, will go to prove, that many of them are not merely double in appearance, but must be allowed to be real binary combinations of two stars, intimately held together by the bond of mutual attraction.

It will be necessary to enter into a certain theory, by which these observations ought to be examined, that we may find to what cause we should attribute such changes in the position, or

distance, of double stars, as will be reported; and, in order to make the required principles very clear, I shall give them in a few short and numbered sentences, that they may be referred to hereafter.

In Plate VII. Fig. 1, let us call the place of the sun, which may also be taken for that of the observer, O. In the centre of an orbit or plane N F S P is α Geminorum; and, if any other star is to be examined, we have only to exchange the letter α for that by which such double star is known. This letter is always understood to represent the largest of the two stars which make up the double star; and a general expression for its smaller companion will be x. N, F, S, P, represent the positions of the different parts of the heavens, with respect to α, north, following, south, and preceding; and the small letters n, f, s, p, stand for the same directions with respect to O. $x \alpha$ P, is the angle of position of the two stars x and α, with the parallel F P.

As the motion of an observer affects the relative situation of objects, we have three bodies to consider, in our investigation of the cause of the changes which will be pointed out; the sun, the large star, and the small star, or, as we have shortly called them, O, α, x. This admits of three cases: a motion of one of the three bodies; another, of two; and a third, of all the three bodies together. We shall now point out the consequences that will arise in each of the cases.

Single Motions.

No. 1. Motion of x. When α and O are at rest, the motion of x may be assumed, so as perfectly to explain any change of the distance of the two stars, and of their angle of position.

No. 2. Motion of α. When x and O are at rest, and α has a motion, either towards P, N, F, or S, then the effect of it, whatever may be the angle P α O, will be had by entering the following Table, with the direction of the given motion.

Motion.	Distance.	Angle.	Quadrants.
αP	− +	+ −	1st and 4th 2 — 3
αF	+ −	− +	1 — 4 2 — 3
αN	− +	− +	1 — 2 3 — 4
αS	+ −	+ −	1 — 2 3 — 4

No. 3. Motion of O. 1st case. When α and x are at rest, and the angle P α O is 90 degrees, a proper motion of O, towards either p, f, n, or s, which will be extremely small when compared with the distance of O from α, can have no effect on the apparent distance, or angle of position, of the two stars; and therefore no other motion, composed of the directions we have mentioned, will induce a change in the comparative situation of α and x.

2d case. When the plane PNFS is oblique to the ray αO, and the angle PαO more than 90 degrees, the effect of the motion of O will be had by the following Table.

Motion.	Distance.	Angle.	Quadrants.
O*p*	+	−	1st and 2d 3 — 4
O*f*	−	+	1 — 2 3 — 4
O*n*	+ −	+ −	1 — 3 2 — 4
O*s*	− +	− +	1 — 3 2 — 4

3d case. When the angle PαO is less than 90 degrees, the following Table must be used.

Motion.	Distance.	Angle.	Quadrants.
O*p*	−	+	1st and 2d 3 — 4
O*f*	+	−	1 — 2 3 — 4
O*n*	− +	− +	1 — 3 2 — 4
O*s*	+ −	+ −	1 — 3 2 — 4

Double Motions.

No. 4. If we admit different motions in two of our three bodies, and if the ratio of the velocities, the directions of the

motions, and the ratio of the distances of the bodies be given quantities, a supposition in which we admit their concurrence, may explain the phenomena of a double star, but can never be probable.

Motions of the three Bodies.

No. 5. If we admit different motions in every one of the three bodies, O, α, x, and if the velocities and directions of the motions, as well as the relative distances of the three bodies are determined, an hypothesis which admits the existence of such motions and situations, may resolve the phenomena of a double star, but cannot have any pretension to probability.

The compass of this Paper will not allow me to give the observations of my double stars at full length; I shall therefore, in the examination of every one of them, only state those particulars which will be required for the purpose of investigating the cause of the changes that have taken place, either in the distance, or angle of position, of the two stars of which the double star is composed.

As the arguments in the case of most of these stars will be nearly the same, it may be expected, that the first two or three which are to be examined will take up a considerable space; and the number of double stars, in which I have already ascertained a change, amounting to more than fifty, it will not be possible to give them all in one paper; I shall therefore confine the present one to a moderate length, and leave it open for a continuation at a future opportunity.

α Geminorum.

From my earliest observations on the distance of the two stars which make up the double star in the head of Castor, given in the first of my catalogues of double stars, we find, that about 23 years and a half ago, they were nearly two diameters of the large star asunder. These observations have been regularly continued, from the year 1778 to the present time, and no alteration in the distance has been perceived: the stars are now still nearly 2 diameters of the large one asunder.

It will be necessary to enter a little into the practicability of ascertaining distances by a method of estimation apparently so little capable of precision. From a number of observations and experiments I have made on the subject, it is certain that the apparent diameter of a star, in a reflecting telescope, depends chiefly upon the four following circumstances: the aperture of the mirror with respect to its focal length; the distinctness of the mirror; the magnifying power; and the state of the atmosphere at the time of observation. By a contraction of the aperture, we can increase the apparent diameter of a star, so as to make it resemble a small planetary disk. If distinctness should be wanting, it is evident that the image of objects will not be sharp and well defined, and that they will consequently appear larger than they ought. The effect of magnifying power is, to occasion a relative increase of the vacancy between two stars that are very near each other; but the ratio of the increase of the distance is not proportional to that of the power, and sooner or later comes to a maximum. The state of the atmosphere is perhaps the most material of the four conditions, as we have it not in our power to alter it. The effects of moisture, damp air, and

haziness, (which have been related in a paper where the causes that often prevent the proper action of mirrors were discussed,) show the reason why the apparent distance of a double star should be affected by a change in the atmosphere. The alteration in the diameter of Arcturus, extending from the first to the last of the ten images of that star, in the plate accompanying the abovementioned paper,* shows a sufficient cause for an increase of the distance of two stars, by a contraction of their apparent disks. A skilful observer, however, will soon know what state of the air is most proper for estimations of this kind. I have occasionally seen the two stars of Castor, from $1\frac{1}{2}$ to 2 and $2\frac{1}{2}$ diameters asunder; but, in a regular settled temperature and clear air, their distance was always the same. The other three causes which affect these estimations, are at our own disposal; an instance of this will be seen in the following trial. I took ten different mirrors of 7 feet focal length, each having an aperture of 6,3 inches, and being charged with an eye-glass which gave the telescope a magnifying power of 460. With these mirrors, one after another, the same evening, I viewed the two stars of our double star; and the result was, that with every one of them, the stars were precisely at an equal distance from each other. These mirrors were all sufficiently good to show minute double stars well; and such a trial will consequently furnish us with a proper criterion, by which we may ascertain the goodness of our telescope, and the clearness of the atmosphere required for these observations. To those who have not been long in the habit of observing double stars, it will be necessary to mention, that, when first seen, they will appear nearer together than after a certain time; nor is it so soon as might be

* See Phil. Trans. for 1803, page 232, Plate III.

expected, that we see them at their greatest distance. I have known it to take up two or three months, before the eye was sufficiently acquainted with the object, to judge with the requisite precision.

Whatever may be the difficulties, or uncertainties, attending the method of determining the distance of two close stars by an estimation of the apparent diameter, it must however be confessed, that we have no other way of obtaining the same end with so much precision. Our present instance of α Geminorum, will show the degree of accuracy of which such estimations are capable, and at the same time prove, that the purpose for which I shall use the estimated interval between the two stars will be sufficiently answered. By an observation of the 10th of May, 1781, we have the diameter of the largest of the two stars to that of the smallest as 6 to 5; and, according to several measures I have taken with the micrometer, we may admit their distance, diameters included, to be five seconds. Then, as the vacancy between the two stars is nearly, but not quite, 2 diameters of the large one, I shall value it at $1\frac{7}{8}$. From this we calculate, that the diameter of the large star, under the circumstances of our estimation, is nearly $1'',35$: so that an error of one quarter of such a diameter, which is the most we can admit, will not exceed $0'',34$. Nor is it of much consequence, if the measure of $5''$ should not be extremely correct; as a small mistake in that quantity will not materially affect the error of estimation by the diameter, which, from what has been said, if the measure was faulty to a second, would not amount to more than one-fifteenth part of it.

Having thus ascertained that no perceptible change in the distance of the stars has taken place, we are now to examine

the angle of position. In the year 1779, it was 32° 47′ north preceding; and, by a mean of the three last measures I have taken, it is now only 10° 53′. In the space of about 23 years and a half, therefore, the angle of position has manifestly undergone a diminution, of no less than 21° 54′; and, that this change has been brought on by a regular and gradual decrease of the angle, will be seen when the rest of the measures come to be examined.

The accuracy of the micrometer which has been used, when the angles of position were taken, being of the utmost importance, it becomes necessary to ascertain how far it will be safe to rely on the result of the measures. It might be easily shown that, in the day time, a given angle, delineated on a card, and stuck up at a convenient distance, may be full as accurately measured by a telescope furnished with this micrometer, as it can be done by any known method, when the card is laid on a table before us; but this would not answer my purpose. For, objects in motion, like the stars, especially when at a distance from the pole, cannot be measured with such steadiness as those which are near us, and at rest. The method of illuminating the wires, and other circumstances, will likewise affect the accuracy of the angles that are measured, especially when the distance of the stars is very small. I shall therefore have recourse to astronomical observations, in order to see what the micrometer has actually done.

January 22, 1802. The position of A Orionis was taken. 1st measure, 52° 38′ south preceding; 2d measure, 54° 14′. Mean of the two measures, 53° 26′. Deviation of the measures from the mean, 48′.

March 4, 1802. 11 Monocerotis. 1st measure, 28° 18′ south

following; 2d measure, 26° 49′. Mean of the two, 27° 34′. Deviation from the mean, 45′.

February 9, 1803. α Geminorum. 1st measure, 6° 11′ north preceding; 2d measure, 4° 48′. Mean of the two, 5° 29′. Deviation from the mean, 41′.

September 6, 1802. η Coronæ. 1st measure, 89° 42′ north following; 2d measure, 89° 38′. Mean of the two, 89° 40′. Deviation from the mean, 2′.

When these observations are considered, we shall not err much if we admit that, in favourable circumstances, and with proper care, the micrometer, by a mean of two measures, will give the position of a double star true to nearly one degree; but, as the opportunities of taking very accurate measures are scarce, it will be necessary to have recourse to some more discordant observations.

February 18, 1803. β Orionis. 1st measure, 72° 58′ south preceding; 2d measure, 67° 24′. Mean of the two, 70° 11′. Deviation from the mean, 2° 47′.

But a memorandum to the observation says, that the evening was not favourable. We may therefore admit, that in the worst circumstances which can be judged proper for measuring at all, an error in the angle of position by two measures will not amount to three degrees.

It will be remarked, when we come to compare single measures which have been taken on different nights, that they are somewhat more discordant; but I have not ventured to reject them on that account, except in cases where it was pretty evident that some mistake in reading off, or other accident to which all astronomical observations are liable, was to be apprehended. Nor can such disagreements materially affect the

conclusions I have drawn, when it appears that the deviations happen sometimes to be on one side, and sometimes on the other side, of the true angle of position. For, since that angle is not a thing that will change in the course of a few nights, the excess of one measure will serve to correct the defect of another; and we are not to think it extraordinary, when stars are so near together, and their motion through the field of view (in consequence of the high magnifying power we are obliged to use) so quick, that we should now and then even fall short of that general accuracy which may be had by a careful use of the micrometer.

I shall now enter into an examination of the cause of the change in the angle of position of the small star near Castor.

A revolving star, it is evident, would explain in a most satisfactory manner, a continual change in the angle of position, without an alteration of the distance. But this, being a circumstance of which we have no precedent, ought not to be admitted without the fullest evidence. It will therefore be right to examine, whether the related phenomena cannot be satisfactorily explained by the proper motions of the stars, or of the sun.

Single Motions.

(a) The three bodies we have to consider, are O, α, and x; and, supposing them to be placed as they were observed to be in the year 1779; the angle $x \alpha$ P, in Fig. 1, will be 32° 47′ north preceding. We are at liberty to let the angle P α O be what will best answer the purpose. Then, in order to examine the various hypotheses that may be formed, according to the arrangement of the principles we have given, we shall begin with No. 1; and, as this admits that all phenomena may be

resolved by a proper motion of x, let us suppose this star to be placed any where far beyond α, but so as to have been seen, in the year 1779, where the angle of position, 32° 47′ north preceding, and the observed distance, near 2 diameters of the large star, required it. With a proper velocity, let it be in motion towards the place where it may now be seen at the same distance from Castor, but under an angle of position only 10° 53′ north preceding. It may then be admitted, that a small decrease of the distance which would happen at the time when the angle of position was 21° 50′, could not have been perceived; so that the gradual change in the observed angle of position, as well as the equality of the distance of the two stars, will be sufficiently accounted for. But the admission of this hypothesis requires, that α Geminorum and the solar system should be at rest; and, by the observations of astronomers, which I shall soon have occasion to mention, neither of these conditions can be conceded.

(b) If, according to No. 2, we admit the motion of α, we shall certainly be more consistent with the observations which astronomers have made on the proper motion of this star;* and, as a motion of the solar system, which I shall have occasion to mention hereafter, has not been rigidly proved, it may, for the sake of argument, be set aside; nor has a proper motion of the star x been any where ascertained. The retrograde annual proper motion of Castor, in right ascension, according to Dr. MASKELYNE, is 0″,105. This, in about $23\frac{1}{2}$ years, during which

* See TOBIÆ MAYERI *Opera inedita. De motu fixarum proprio,* page 80. Also Dr. MASKELYNE's first Volume of Observations. Explanation and Use of the Tables, page iv. Or Mr. WOLLASTON's Astronomical Catalogue, end of the Preface. Likewise *Connoissance des Temps pour l'Année* VI. page 203. *Sur le Mouvement particulier propre a différentes Etoiles*; par Mons. DE LA LANDE.

time I have taken notice of the angle of position and distance of the small star, will amount to a change of nearly 2″,47. Then, if we enter the short Table I have given in No. 2, with the motion αP, we find, that in the first quadrant, where the small star is placed, the distance between the two stars will be diminished, and the angle of position increased. But since it appears, by my observations, that the distance of the stars is not less now than it was in 1780; and that, instead of an increase in the angle of position, it has actually undergone a diminution of nearly 22 degrees; it follows, that the motion of α Geminorum in right ascension, will not explain the observed alterations in the situation of this double star. If, according to Mr. DE LA LANDE's account,* we should also consider the annual proper motion of α in declination, which is given 0″,12 towards the north, we shall find, by entering our Table with the motion αN, amounting to 2″82, that the distance of the two stars will be still more diminished; but that, on the contrary, the angle of position will be much lessened; and, by combining the two motions together, the apparent disks of the two stars should now be a little more than one-tenth of a second from each other, and the angle of position 35 degrees south preceding. But, since neither of these effects have taken place, the hypothesis cannot be admitted.

(c) That the sun has a proper motion in space, I have shown with a very high degree of evidence, in a paper which was read at the Royal Society about twenty years ago.† The same opinion was before, but only from theoretical principles, hinted at by Mr. DE LA LANDE, and also by the late Dr. WILSON, of

* See page 211 of the treatise before referred to.

† See Phil. Trans. Vol. LXXIII. page 247.

Glasgow;* and has, since the publication of my paper, been taken up by several astronomers,† who agree that such a motion exists. In consequence of this, let us now, according to No. 3, assign to the sun a motion in space, of a certain velocity and direction. Admitting therefore α and x to be at rest, let the angle $P\alpha O$ be 90 degrees; then, by the 1st case of No. 3, we find that none of the observed changes of the angles of position will admit of an explanation. There is moreover an evident concession of the point in question, in the very supposition of the above angle of 90 degrees; for, if x be at the same distance as α from the sun, and no more than $5''$ from that star, its real distance, compared to that of the sun from the star, will be known; and, since that must be less than the 40 thousandth part of our distance from Castor, these two stars must necessarily be within the reach of each other's attraction, and form a binary system.

(d) Let us now take the advantage held out by the 2d case of No. 3, which allows us to place x far behind α; in which situation, the angle $P\alpha O$ will be more than 90 degrees. The star x being less than α, renders this hypothesis the more plausible. Now, as a motion of Castor, be it real or apparent, has actually been ascertained, we cannot set it aside; the real motion of O, therefore, in order to account for the apparent one of α, must be of equal velocity, and in a contrary direction; that is, when decomposed, $0'',105$ towards *f*, and $0'',12$, towards *s*. The effect of the sun's moving from O towards *f*, according to

* See my note in Phil. Trans. Vol. LXXIII. page 283.

† See *Astronomisches Jahrbuch für das Jahr* 1786; *seite* 259. *Uber die fortrückung unseres Sonnen-Systems, von* HERRN *Professor* PREVOST. *Und für das Jahr* 1805; *seite* 113.

the 1st Table in No. 3, is, that the distance between the two stars will be diminished, and the angle of position increased. But these are both contrary to the observations I have given. The motion of O in declination towards *s*, according to the same Table, will still diminish the distance of the two stars, but will also diminish the angle of position. Then, since a motion in right ascension increases the angle, while that in declination diminishes it, the small star may be placed at such a distance that the difference in the parallax, arising from the solar motion, shall bring the angle of position, in $23\frac{1}{2}$ years, from 32° 47′ to 10° 53′; which will explain the observed change of that angle. The distance of the star *x*, for this purpose, must be above $2\frac{1}{3}$ times as much as that of *α* from us. But, after having in this manner accounted for the alteration of the angle of position, we are, in the next place, to examine the effect which such a difference of parallax must produce in the apparent distance of the two stars from each other. By a graphical method, which is quite sufficient for our purpose, it appears, that the union of the two motions in right ascension and declination, must have brought the two stars so near, as to be only about half a diameter of the large star from each other; or, to express the same in measures, the centres of the stars must now be 1″,8 nearer than they were $23\frac{1}{2}$ years ago. But this my observations cannot allow; for we have already shown, that any change of more than 3 or 4-tenths of a second must have been perceived.

If, on the other hand, we place the star *x* at such a distance that the solar parallax may only bring it about 4-tenths of a second nearer to *α*, which is a quantity we may suppose to have escaped our notice in estimating the apparent distance of the two stars, then will the angle of position be above 20 degrees

too large. This shows, that no distance, beyond Castor, at which we can place the star, will explain the given observations.

(e) The last remaining trial we have to examine, is to suppose x to be nearer than α; the angle $P\alpha O$, will then be less than 90 degrees; and the effect of a motion of O towards f, by the 2d Table in No. 3, will be an increase of the distance of the two stars, and a diminution of their angle of position. But the motion Os, which is also to be considered, will add to the increase of the distance, and counteract the diminution of the angle. It is therefore to be examined, whether such an increase of distance as we can allow to have escaped observation, will explain the change which we know to have happened in the angle, during the last $23\frac{1}{2}$ years. By the same method of compounding the two motions as before, it immediately appears, than we cannot place the small star more than about 1-tenth of the distance $O\alpha$ on this side of Castor, without occasioning such an increase of the apparent distance of the two stars as cannot possibly be admitted; and that, even then, the angle of position, instead of being less, will be a few degrees larger, at the end of $23\frac{1}{2}$ years, than it was at the beginning. This hypothesis, therefore, like all the foregoing ones, must also be given up, as inconsistent with my observations.

It is moreover evident, that the observations of astronomers on the proper motion of the stars in general, will not permit us to assume the solar motion at pleasure, merely for the sake of accounting for the changes which have happened in the appearances of a double star. The proper motion of Castor, therefore, cannot be intirely ascribed to a contrary motion of the sun. For we can assign no reason why the proper motion of this star alone, in preference, for instance, to that of Arcturus,

of Sirius, and of many others, should be supposed to arise from a motion of the solar system. Now, if they are all equally intitled to partake of this motion, we can only admit it in such a direction, and of such a velocity, as will satisfy the mean direction and velocity of the general proper motions of the stars; and place all deviations to the account of a real proper motion in each star separately.

Double Motion.

(f) In order to explain the phenomena of our double star, according to No. 4, by the motion of two bodies, for instance α and x, it will be required that they both should move in given directions; that the velocities of their motions should be in a given ratio to each other; and that this ratio should be compounded with the ratio of their distances from O; a supposition which must certainly be highly improbable. To show this with sufficient evidence, let us admit that, according to the best authorities, the annual proper motion of Castor is $-0'',105$ in right ascension, and $0'',12$ in declination towards the north. Then, as the small star, without changing its distance, has moved through an angle of $21° \, 54'$, the only difference in the two motions of these stars, will be expressed by the extent of the chord of that angle. To produce the required effect, it is therefore necessary that the motion of α, which is given, should regulate that of the small star, whose relative place at the end of $23\frac{1}{2}$ years is also given. Then, as α moves in an angle of $53° \, 31'$ north preceding, and with a velocity which, being expressed by the space it would describe in $23\frac{1}{2}$ years, will be $3'',51$, it is required that x shall move in an angle of $29° \, 25'$, likewise north preceding, and with a velocity of $3'',02$. The

ratio of the velocities, therefore, and the directions of the motions, are equally given. But this will not be sufficient for the purpose: their distance from O must also be taken into consideration. It has been shown, that the two stars cannot be at an equal distance from us, without an evident connection; it will therefore be necessary for those who will not allow this connection, to place one of them nearer to us than the other. But, as the motions which have been assumed, when seen from different distances, will subtend lines whose apparent magnitudes will be in the inverse ratio of the assumed distances, it is evident that this ratio, if the motions are given, must also be a given one; or that, if the distances be assumed, the ratio of the motions must be compounded with the ratio of the distances. How then can it be expected that such precise conditions should be made good, by a concurrence of circumstances owing to mere chance? Indeed, if we were inclined to pass by the difficulties we have considered, there is still a point left which cannot be set aside. The motion of the solar system, although its precise direction and velocity may still be unknown, can hardly admit of a doubt; we have therefore a third motion to add to the former two, which consequently will bring the case under the statement contained in our 7th number, and will be considered hereafter.

(g) If we should intend to change our ground, and place the two motions in O and x, it will then be conceded, that the motion of α is only an apparent one, which owes its existence to the real motion of the sun. By this, the effect of the solar parallax on any star at the same distance will be given; and it cannot be difficult to assume a motion in x, which shall, with the effect of this given parallax, produce the apparent motion, in the

direction of a chord from the first to the last angle of position pointed out by my observations; taking care, however, not to place the stars α and x at the same distance from us; and using the inverse ratio of the solar parallax as a multiple in the assigned motion. For instance, let the sun have a motion of the velocity expressed as before by $3'',51$, and in a direction which makes an angle of $53^\circ\ 31'$ south following with the parallel of α Geminorum; and let the small star x have a real motion in an angle of $18^\circ\ 40'$ south preceding from the parallel of its situation, and with a real velocity which, were it at the distance of α, would carry it through $1'',89$. Then, if the distance of the small star be to that of the large one as 3 to 2, the effect of the solar parallax upon it will be $\frac{2}{3}$ of its effect upon α; that is, while α, which is at rest, appears to move over a space of $3'',51$, in an angle of $53^\circ\ 31'$ north preceding, the parallactic change of place in x will be $2'',34$ in the same direction. This, though only an apparent motion, will be compounded with the real motion we have assigned to it, but which, at the distance of α, will only appear as $1'',26$; and the joint effect of both will bring the star from the place in which it was seen $23\frac{1}{2}$ years ago, to that where now we find it situated. α, in the same time, will appear to have had an annual proper motion of $-\ 0'',105$ in right ascension, and $0'',12$ in declination towards the north; and thus all phenomena will be explained.

From this statement, we may draw a consequence of considerable importance. If we succeed, in this manner, in accounting for the changes observed in the relative situation of the two stars of a double star, we shall fail in proving them to form a binary system; but, in lieu of it, we shall gain two other points, of equal value to astronomers. For, as α Geminorum, according

to the foregoing hypothesis, is a star that has no real motion, its apparent motion will give us the velocity and direction of the motion of the solar system; and, this being obtained, we shall also have the relative parallax of every star, not having a proper motion, which is affected by the solar motion. Astronomical observations on the proper motion of many different stars, however, will not allow us to account for the motion of α Geminorum in the manner which the foregoing instance requires; the hypothesis, therefore, of its being at rest, must be rejected.

(*h*) If we place our two motions in O and α, we shall be led to the same conclusion as in the last hypothesis. The known proper motion of α, and the situations of the small star in 1779 and 1803, given by my observations, will ascertain the apparent motion of x, now supposed to be at rest. Then, since the change in the place of x must be intirely owing to the effect of parallax, it will consequently give us, in the same manner as before, the quantity and direction of the motion of the solar system, and the relative distances of all such stars as are affected by it. But, here again, the solar motion required for the purpose is such as cannot be admitted; and the hypothesis is not maintainable.

Motion of the three Bodies.

(*i*) There is now but one case more to consider, which is, according to No. 5, to assign real motions to all our three bodies; and this may be done as follows. Suppose the sun to move towards λ Herculis, with the annual velocity 1.

Let the apparent motion of α Geminorum be as it is stated in the astronomical tables before mentioned; but suppose it to arise from a composition of its real motion with the effect of the

systematical parallax, as we may call that apparent change of place of stars which is owing to the motion of the solar system. Let the real motion of x, aided by the effect of the same parallax, be the cause of the changes in the angle of position which my observations have given. We may admit the largest of the two stars of our double star to be of the second magnitude; and, as we are not to place x too near α, we may suppose its distance from O to be to that of α from the same as 3 to 2. In this case, O will move from the parallel of α, in an angle of 60° 37′ north following, with an apparent annual velocity of ,4536. The motion of α in right ascension, may be intirely ascribed to solar parallax; but its change of declination, cannot be accounted for in the same manner. Let us therefore admit that the solar velocity, in the direction we have calculated, will produce an apparent retrograde motion in α, which, in $23\frac{1}{2}$ years will amount to 2″,085 in right ascension. But the same parallax will also occasion a change in declination, towards the south preceding, of 3″,701; and, as this will not agree with the observed motion of α, we must account for it by a proper motion of this star directly towards the north. The real annual velocity required for this purpose, must be 1,3925.

The apparent motion of x, by parallax, at the distance we have placed this star, will be 2″,832 towards the south preceding; and, by assigning to it an annual proper motion of the velocity 1,3354, in the direction of 73° 10′ north preceding its own parallel, the effect of the solar parallax and this proper motion together, will have caused the small star, in appearance, to revolve round α, so as to have produced all the changes in the angle of position which my observations have given; and, at the same time, α will have been seen to move from its former

place, at the annual rate of $0'',105$ in right ascension, and $0'',12$ in declination towards the north.

In this manner, we may certainly account for the phenomena of the changes which have taken place with the two stars of α Geminorum. But the complicated requisites of the motions which have been exposed to our view, must surely compel every one who considers them to acknowledge, that such a combination of circumstances involves the highest degree of improbability in the accomplishment of its conditions. On the other hand, when a most simple and satisfactory explanation of the same phenomena may be had by the effects of mutual attraction, which will support the moving bodies in a permanent system of revolution round a common centre of gravity, while at the same time they follow the direction of a proper motion which this centre may have in space, it will hardly be possible to entertain a doubt to which hypothesis we ought to give the preference.

As I have now allowed, and even shown, the possibility that the phenomena of the double star Castor may be explained by proper motions, it will appear that, notwithstanding my foregoing arguments in favour of binary systems, it was necessary, on a former occasion, to express myself in a conditional manner,* when, after having announced the contents of this Paper, I added, "*should these observations be found sufficiently conclusive;*" for, if there should be astronomers who would rather explain the phenomena of a small star appearing to revolve round Castor by the hypothesis we have last examined, they may certainly claim the right of assenting to what appears to them most probable.

I shall now enter into a more detailed examination of the

* See Phil. Trans. for 1802, page 486.

several angles of position I have taken at different times, and show that they agree perfectly well with the appearances which must arise from the revolution of a small star round Castor. A calculation of these angles may be had, by finding the annual motion of the small star, from the change of 21° 54′, which has been shown to have taken place in 23 years and 142 days. Accordingly, I have given, in the 1st column of the following Table, the time when the angles were taken. In the 2d, are the angles as they were found by measure; they are all in the north-preceding quadrant. The 3d column contains a calculation from the annual motion of 56′,18, obtained as before mentioned: it shows what these angles should have been, according to our present supposition of a revolving star. And the last column gives the difference between the observed and calculated angles.

Times of the observations.	Observed angles.	Calculated angles.	Differences.
Nov 5, 1779 - -	32° 47′	32° 47′	0° 0′
Feb. 23, 1791 - -	22 57	22 11	+ 0 46
Feb. 26, 1792 - -	27 16	21 16	+ 6 0
Dec. 15, 1795 - -	13 52	17 42	— 3 50
March 26, 1800 - -	18 8	13 41	+ 4 27
April 23, 1800 - -	10 30	13 37	— 3 7
Dec. 31, 1801 - -	7 58	12 2	— 4 4
Jan. 10, 1802 - -	10 53	12 1	— 1 8
Jan. 23, 1802 - -	10 28	11 59	— 1 31
Feb. 28, 1802 - -	13 0	11 53	+ 1 7
Feb. 11, 1803 - -	7 53	11 0	— 3 7
March 23, 1803 - -	13 23	10 54	+ 2 29
March 27, 1803 - -	10 53	10 53	0 0

On looking over the 4th column of this table, it will be found, that the differences between the observed and calculated angles are not greater than may be expected, considering that most of the early measures are single, and cannot have the accuracy which may be obtained by repetition. Even as they are, we must acknowledge them sufficient to ascertain the gradual change in the angle of position of the two stars. In one place, the difference amounts to six degrees; but it will soon appear, that a more accurate annual motion gives a calculated position which takes off much of the error of this measure.

In a conversation with my highly esteemed friend the Astronomer Royal, he happened some time ago accidentally to mention, that Dr. BRADLEY had formerly observed the two stars of α Geminorum to stand in the same direction with Castor and Pollux. It occurred to me immediately, that if the time of this observation could be nearly ascertained, it would be of the greatest importance to the subject at present under consideration. For, should Dr. BRADLEY's position be very different from a calculated one, it would induce us at once to give up the idea of a revolving star. The observation was made by Dr. BRADLEY with a view to see whether any change could be perceived in the course of the year, by which the annual parallax of the stars might be discovered. Dr. MASKELYNE, who had this information from Dr. BRADLEY in conversation, had made a memorandum of it in his papers. He has been so kind as to look for it; and, as soon as he found the note, he sent me the following copy, which I have his permission to transcribe.

"*Double star Castor. No change of position in the two Stars:*
"*the line joining them, at all times of the year, parallel to the line*

"*joining Castor and Pollux in the heavens, seen by the naked* "*eye.*"

Dr. MASKELYNE informs me, that the observation must have been made about the year 1759; and also mentions, that he himself verified the fact, as to the line joining the two stars appearing through the telescope parallel to the line joining Castor and Pollux, in 1760 or 1761; but that he did not examine it at various times of the year.

The advantage of having an angle of position observed in 1759 by Dr. BRADLEY, and so soon after verified by Dr. MASKELYNE, will give us an addition of 20 years to our period. On calculating the right ascension and polar distance of Castor and Pollux for November 5, 1759, it appears, that a line drawn from Pollux through Castor, must have made an angle of 56° 32′ north preceding with the parallel of that star; and, this being also the position of our double star, we have an interval of 43 years and 142 days, for a change of 45° 39′, from the time of Dr. BRADLEY's observation to that of my last measure of the angle. By this we are now enabled to correct our former calculation, which was founded upon a supposition that the first angle of position I had taken was perfect; but this could hardly be expected, and on examination it appears that the measure was 2° 40′ too little. The annual motion, by our increased period, is 1° 3′,1; and the computation of the angles of position in the 3d column of the following Table, as well as the differences contained in the 4th, are made according to this motion.

Times of the observations.	Observed angles.	Calculated angles.	Differences.
Nov. 5, 1759 - -	56° 32′	56° 32′	0° 0′
Nov. 5, 1779 - -	32 47	35 29	— 2 42
Feb. 23, 1791 - - -	22 57	23 36	— 0 39
Feb. 26, 1792 - -	27 16	22 32	+ 4 44
Dec. 15, 1795 - -	13 52	18 32	— 4 40
March 26, 1800 - -	18 8	14 3	+ 4 5
April 23, 1800 - - -	10 30	13 58	— 3 28
Dec. 31, 1801 - -	7 58	12 12	— 4 14
Jan. 10, 1802 - - -	10 53	12 10	— 1 17
Jan. 23, 1802 - -	10 28	12 7	— 1 39
Feb. 28, 1802 - - -	13 0	12 1	+ 0 59
Feb. 11, 1803 - -	7 53	11 1	— 3 8
March 23, 1803 - - -	13 23	10 54	+ 2 29
March 27, 1803 - -	10 53	10 53	0 0

When the result of this Table is compared with that of the former, it will be seen that my observations agree not only very well with Dr. Bradley's position, but even give more equally divided differences than before, so that the excess and differences counteract each other better than in the first Table.

The time of a periodical revolution may now be calculated from the arch of 45° 39′, which has been described in 43 years and 142 days. The regularity of the motion gives us great reason to conclude, that the orbit in which the small star moves about Castor, or rather, the orbits in which they both move round their common centre of gravity, are nearly circular, and at right angles to the line in which we see them. If this should be nearly true, it follows, that the time of a whole apparent

revolution of the small star round Castor, will be about 342 years and two months.

γ Leonis.

Our foregoing discussions will greatly abridge the arguments which may be used, to show that this star and its small companion are also probably united in forming a binary system. But, in order to give more clearness to our disquisition, we shall follow the arrangement which has been used with α Geminorum, and prefix the same letters to our paragraphs. Then, if any one article should appear to be not sufficiently explained, we need but turn back to our first double star, where the same letter will point out what has already been said more at large on the subject; and an application of it may easily be made.

The distance of the stars γ and x, as I shall again call the small one, has undergone a visible alteration in the last 21 years. The result of a great number of observations on the vacancy between the two stars, made with the magnifying powers of 278, 460, 657, 840, 932, 1504, 2010, 2589, 3168, 4294, 5489, and 6652, is, that with the standard power and aperture of the 7-feet telescope, the interval in 1782 was $\frac{1}{4}$ of a diameter of the small star, and is now $\frac{3}{4}$. With the same telescope, and a power of 2010, it was formerly $\frac{1}{2}$ of a diameter of the small star, and is now full 1 diameter. In the years 1795, 1796, and 1798, the interval was found to have gradually increased; and all observations conspire to prove, that the stars are now $\frac{1}{2}$ a diameter of the small one farther asunder than they were formerly. The proportion of the diameter of γ to that of x, I have, by many observations, estimated as 5 to 4.

The first measured angle in 1782, is 7° 37′ north following;*

* In my second Catalogue of double Stars, (Phil. Trans. for 1785, page 48,) the

and the last, which has been lately taken, is 6° 21′ south following. The sum of these angles gives 13° 58′, for the change that has taken place in 21 years and 38 days. To account for this, we are to have recourse, as before, to the various motions of the three bodies.

Single Motions.

(*a*) The motion of x alone cannot be admitted, since it is known that γ Leonis is not at rest. The annual proper motion of this star, according to M. DE LA LANDE, is + 0″,38 in right ascension, and 0″,04 in declination towards the south.

(*b*) γ cannot be the only moving body; because its motion in right ascension only, which, in 21,1 years, at the parallel of γ, amounts to 7″,49, would have long ago taken it away from the small star.

(*c, d, e,*) The sun cannot be the only moving body; because its motion in right ascension will not account for that of γ Leonis, which star therefore cannot be at rest. And, if we were willing to give up the former assumed solar motion, in order to fix upon such a one as would explain the motion of γ, we should be under a necessity to contradict the united evidence of the proper motions of many principal stars which are in opposition to it.

Double Motions.

(*f*) When two motions are proposed, we cannot fix upon γ and x for the moving bodies, unless we should set aside the solar motion, and this, we know, cannot properly admit of a doubt.

angle of position is 5° 24′. This was taken April 18, 1783; and, not being acquainted with the motion of the small star, I supposed it to be more accurate than the former measure.

(g) That we cannot allow O and x to be the two bodies in motion, follows from the insufficiency of the solar motion to account for that of γ, which must be real, or at least partly so.

(h) If O and γ are the moving bodies, the given situations of x, in the years 1782 and 1783, point out an apparent motion of x, which must be intirely owing to the solar parallax; and, therefore, those who will admit this hypothesis, must grant the discovery of the motion of the solar system, and of the proportional parallax of the two stars γ and x. Let us however examine whether any motion of the sun, such as we can admit, will account for the change of position and distance pointed out by my observations of the small star near γ Leonis.

The joint effect of proper motion and parallax, has carried γ from its situation in 1782 to that where we now find it. The small star, having all this time, in appearance, accompanied γ, must have gone through a space of $7'',98$, in a direction which makes an angle of $8° 30'$ south following with the parallel of γ, in order to be at its present distance from it, and at the same time to have undergone the required change of its angle of position. Now, as the supposition we are examining requires this small star to be actually at rest, it will be necessary to assign to the sun an opposite motion of the same velocity, in order to make that of x only an apparent one. The consequence of this will be a retrograde motion of the sun, which it is well known cannot be admitted.

Motion of the three Bodies.

(i) A motion of all the three bodies, is the only way left to explain the phenomena of our double star; and I shall now again point out the very particular circumstances which it is

requisite should all happen together, to produce the intended effect.

Let the motion of the sun, with the same annual velocity 1, as in the case of α Geminorum, be directed towards λ Herculis. Then the effect of this motion will show itself at the place of γ Leonis, in the annual velocity of ,3314, and in a direction which makes an angle of 31° 11′ south preceding with the parallel of that star. In this calculation, I have admitted the distance of the largest of the two stars of γ from the sun to be 3, that of α Geminorum being 2. But, if any other distance should hereafter be considered as more probable, the calculation may be easily adapted to it. The consequence of the parallax thus produced on γ Leonis in 21,1 years, will be an apparent motion of 2″,788 south preceding, in the abovementioned direction; and, on x, it will be in the same time, and in the same direction, 1″,091. As the small star must not be too near γ, we have, in the calculation, supposed it to be at the distance of 4 from O.

The real annual proper motion of γ is required to be 3,5202; and its direction must make an angle of 3° 40′ north following with the parallel. By this motion alone, γ would have passed over a space of 9″87 in 21,1 years; but, when it is combined with the apparent motion arising from parallax, the star will come into its present situation.

The real annual motion of x must be 4,6294, in a direction 0° 20′ south following. This will carry it over 9″,74, in 21,1 years; and, when combined with the apparent motion which the solar parallax will occasion, both together will bring it to its proper distance from γ Leonis, and to a situation which will agree with the last observed angle of position.

From what has been said, it is again evident, that not only as

many particular circumstances must concur in explaining the phenomena of γ Leonis as we have pointed out with α Geminorum, but that a very marked condition is added in our second double star, which requires an adjustment of velocities in γ and x, which shall also fit the same solar motion that was used in α Geminorum. And this proves, that every additional double star which requires the same condition in order to have its appearances explained, will inforce the arguments which have been used, in a compound ratio.

If, on the other hand, we have recourse to the simplicity of the known effects of attraction, and admit the two stars of our present double star to be united in one system, all the foregoing difficulties of accounting for the observed phenomena will vanish. Whatever may be the proper motion of the sun, the parallax arising from that cause will affect both stars equally, on account of their equal distance from the sun. The proper motion of γ Leonis also may be in any direction, and of any given velocity, such as will agree best with astronomical observations; since the motion of a system of bodies will not interfere with the particular motion of the bodies that belong to it, so that our secondary star will continue its revolution round the primary one without disturbance.

It will now be necessary to examine the observed angles of position, and to compare them with calculated ones; but, as there has been a change in the distance of the two stars, it is evident that, if they revolve in circular orbits, the situation of the plane of their revolution must be considerably inclined to the line in which we see the principal star.

Let N F S P, Fig. 2, be the orbit in which x revolves about γ placed in the centre. Suppose a perpendicular to be erected at

γ leading to O, not expressed in the figure. By an observation of Feb. 16, 1782, we have the angle $F \gamma x = 7° 37'$ north following; and the proportion of the apparent diameter of γ to that of x has been given as 5 to 4. It has also been ascertained, that the vacancy between the apparent diameters, when the first angle of position was taken, was $\frac{1}{4}$ diameter of the small star; and the last angle of position being $6° 21'$ south following, with a distance between the stars of $\frac{3}{4}$ diameter of the small star, we obtain the two points or centres of the small stars $x x'$, through which an ellipsis $a b x x' c d$ may be drawn about γ. This will be the apparent orbit in which the small star will be seen to move about γ, by an eye placed at O. And the inclination of the orbit to the line in which we see the double star, will be had sufficiently accurate to enable us to give a calculation of the several angles of position that have been taken. The ellipsis we have delineated shows that the small star, in its first situation x, could not be much past its conjunction at b, and that, consequently, in passing from x to x', the parts of the apparent elliptical arch, which are projections of the real circular arch $b b'$, would be described in times nearly proportional to the time in which the whole arch has been described. Upon these principles, the 3d column of the following Table has been calculated.

Times of the observations.	Observed angles.	Calculated angles.	Differences.
Feb. 16, 1782 - -	7° 37′ *nf*	7° 37′	0° 0′
April 18, 1783 - -	5 24 *nf*	6 51	− 1 27
Jan. 24, 1800 - -	3 16 *sf*	4 15	− 0 59
Feb. 19, 1800 - -	3 23 *sf*	4 18	− 0 55
March 26, 1800 - -	3 47 *sf*	4 22	− 0 35
Jan. 26, 1802 - -	6 4 *sf*	5 35	+ 0 29
Feb. 10, 1803 - -	3 33 *sf*	6 16	− 2 43
March 22, 1803 - -	6 32 *sf*	6 20	+ 0 12
March 26, 1803 - -	6 21 *sf*	6 21	0 0

The difference between the calculated and observed angles, contained in the 4th column of the preceding Table, is so little, that we may look upon the gradual change of these angles as established by observation; and we may form a calculated estimate of the time which will be taken up by the mutual revolution of the two stars. The apparent places $x\,x'$, being referred to their real ones, give the arch bb', which has been described in 21 years and 38 days; and this arch, seen from the centre γ, is about 6° 20′: it follows, that the length of a whole revolution of our small star round γ Leonis, will be about 1200 years.

ε *Bootis.*

This beautiful double star, on account of the different colours of the stars of which it is composed, has much the appearance of a planet and its satellite, both shining with innate but differently coloured light.

There has been a very gradual change in the distance of the two stars; and the result of more than 120 observations, with

different powers, is, that with the standard magnifier, 460, and the aperture of 6,3 inches, the vacancy between the two stars, in the year 1781, was $1\frac{1}{2}$ diameter of the large star, and that it now is $1\frac{3}{4}$. By some earlier observations, the vacancy was found to be considerably less in 1779 and 1780; but the 7-feet mirror then in use was not so perfect as it should have been, for the purpose of such delicate observations. By many estimations of the apparent size of the stars, I have fixed the proportion of the diameter of ε to that of x, as 3 to 2. August 31, 1780, the first angle of position measured 32° 19′ north preceding;* and, March 16, 1803, I found it 44° 52′, also north preceding: the motion, therefore, in 22 years and 207 days, is 12° 33′. It should also be noticed, that while the apparent motion of α Geminorum, and of γ Leonis, is retrograde, that of ε Bootis is direct.

A proper motion in this star, if it has any, is still unknown; our former arguments, therefore, cannot be applied to it, without some additional considerations; and, as many others of my double stars will stand in the same predicament, I shall give an outline of what may be said, to show that this, and probably many of the rest, are also binary systems.

Single Motions.

(*a*—*e*) If ε Bootis is a star in which no proper motion can be perceived, we may infer, from the highly probable motion of the solar system, that this star, which is of the 3d magnitude, and on that account within the reach of parallax, must have a real motion, to keep up with the sun, in order to prevent an

* The angle of position, in my first Catalogue of double Stars, Phil. Trans. for 1782, page 115, is 31° 34′ (it should be 54′) north preceding. This will be found to be a mean of the three first measures hereafter given in a Table of positions.

apparent change of place, which must otherwise have happened. In this case, no single motion can be admitted to explain the phenomena of our double star. But, if a real proper motion of ε Bootis should hereafter be ascertained, the arguments we have used in the case of γ Leonis, will lead to the same conclusion.

Double Motions.

(f) ε and x cannot be the moving bodies; and our former argument (f) will apply to every double star whatsoever.

(g) O and x cannot be alone in motion; for, if no motion in ε can be perceived, it must move in a similar manner with the sun, and none of the three bodies will be at rest. But, if its proper motion shall hereafter be found out, it must either be exactly the reverse of the solar motion, and therefore only an apparent one, or it will be more or less different. In the latter case, all the three bodies must be in motion; in the former, the exact quantity of the solar motion will be discovered, and the relative parallax of many stars may be had by observation.

(h) If O and ε are the two bodies in motion, and if at the same time no motion in ε can be perceived, then the apparent motion of x must be intirely owing to the different effect of the solar parallax on ε and x; but the effect of the solar parallax on x, can only be in a direction contrary to the motion of the sun, which, being north following the small star, whether it be nearer or farther from us than ε, must have an apparent motion towards the south preceding part of the heavens. But this is directly in opposition to my observation of the motion of the small star, which, these last 23 years, has been directed towards the north following.

Motion of the three Bodies.

(i) Let the motion of the sun be again towards λ Herculis; then, if no motion in ε Bootis be perceivable, it must move exactly like O. Highly improbable as it is, let it be admitted. Then, in addition to this extraordinary supposition, a third motion is also required for x, which, aided by the solar parallax, is to carry it likewise within a quarter of a diameter of ε, into the same place where, though unperceived, the large star has been carried by its own motion; that is, in order to be apparently at rest, the sun, ε Bootis, and its small companion, must all move exactly alike, setting aside the very little difference in the position and distance of the small star, which, in the whole, amounts to little more than 6-tenths of a second; than which, certainly nothing can be more improbable.

But, if ε shall hereafter be found not to have been at rest during the time of my observations upon it, then its place will be given; and, since also the situation of x, with respect to ε, is to be had from my angles of position and distances of the two stars, the case will be similar to that which has already been considered, in the paragraph (i), under the head of γ Leonis.

I may here add a remark with regard to ε Bootis, which will be applicable to several more of my double stars. In the milky-way, a multitude of small stars are profusely scattered, and their arrangement is very different from what we perceive in those parts of the heavens which are at a considerable distance from it. About ε Bootis, which is situated in what I have formerly called figuratively a nebulous part of the heavens,* there are, comparatively speaking, hardly any stars; and, that so

* See Phil. Trans. for 1784, page 449.

remarkable a star as ε should have a companion, seems almost to amount to a proof that this very companion is, as it appears to be, a connected star. The *onus probandi,* therefore, ought in justice to fall to the share of those who would deny the truth of what we may call a fact; and I believe the utmost they could do, would be to prove that we may be deceived; but they cannot show that this star has no connection with ε Bootis.

This argument will be much supported, when we consider that many of the double stars in the milky-way are probably such as have one of the scattered stars, nearly in the same line, at a great distance behind them. In this case, the two stars of the double star have no connection with each other; and the great number of them in the milky-way, is itself an indication of this effect of the scattered multitude of small stars. In the single constellation of Orion, for instance, we have no less than 43, pointed out by my catalogues; ten of which are of the first class, and yet have undergone no change of distance or position since I first perceived them. But, with apparently insulated stars, such as ε Bootis, the case is just the reverse.

If, in consequence of our former arguments, and the present remarks, we place ε Bootis among the stars which hold a smaller one in combination, we may delineate its orbit as in Plate VIII. Fig. 3.

Let PNFS represent a circle, projected into the elliptical orbit $axx'bcd$. ε is the large star; and xx' are the first and last measured north preceding situations of the small one, as given in the following Table.

Times of the observations.	Observed angles.	Calculated angles.	Differences.
August 31, 1780 -	32° 19′	33° 58′	− 1° 39′
March 13, 1781 - -	30 21	34 13	− 3 52
May 10, 1781 - -	33 1	34 18	− 1 17
Feb. 17, 1782 - -	38 26	34 40	+ 3 46
August 18, 1796 -	45 32	41 40	+ 3 52
Jan. 28, 1802 - -	49 18	44 19	+ 4 59
August 31, 1802 - -	46 47	44 36	+ 2 11
March 23, 1803 -	43 43	44 52	− 1 9
March 26, 1803 - -	44 52	44 52	0 0

The real motion from b to b' is projected into that from x to x'; and, while the elliptical arch subtends an angle of 12° 33′, the circular one will be about 4° 50′.

From the figure of the orbit, we may conclude that the small star, in its first position, at x or b, was not more than between 30 and 40 years past its conjunction; and that, consequently, the parts of the arch xx', were nearly proportional to the times of their being described. The positions have been calculated upon this principle; but with some allowance for the first observed angle, which I suppose to have been a little too small; and, though the differences of the observed and calculated angles are pretty considerable, the observations are still sufficiently consistent to prove the gradual change of the situation of the small star.

The quantity of the change in 22 years and 207 days, will show that a periodical revolution cannot take up less than 1681 years. The real figure and situation of the orbit, with many other particulars, are still unknown; it is, therefore, unnecessary

to point out the uncertainties in which the investigation of the periodical time of the small star about ε Bootis must long remain involved.

ζ Herculis.

My observations of this star furnish us with a phenomenon which is new in astronomy; it is, the occultation of one star by another. This epoch, whatever be the cause of it, will be equally remarkable, whether owing to solar parallax, proper motion, or motion in an orbit whose plane is nearly coincident with the visual ray. My first view of this star, as being double, was July 18, 1782. With 460, the stars were then $\frac{1}{2}$ diameter of the small star asunder. The large star is of a beautiful bluish white; and the small one ash-coloured.

July 21, of the same year, I measured the angle of position, 20° 42′ north following. With the standard power, the distance of the stars remained as before. With 987, they were one full diameter of the small one asunder.

In the year 1795, I found it difficult to perceive the small star; however, in October of the same year, I saw it plainly double, with 460; and its position was north following.

Other business prevented my attending to this star till the year 1802, when I could no longer perceive the small star. Sometimes, however, I suspected it to be still partly visible; and, in September of the same year, with 460, the night being very clear, the apparent disk of ζ Herculis seemed to be a little lengthened one way. With the 10-feet telescope, and a power of 600, I saw the two stars of η Coronæ very distinctly; and, having in this manner proved the instrument to act well, I directed it to ζ Herculis, and found it to have the appearance of a

lengthened, or rather wedge-formed star; after which, I took a measure of the position of the wedge.

Our temperature is seldom uniform enough to permit the use of very high powers; however, on the 11th of April, 1803, I examined the apparent disk, with a magnifier of 2140, and found it, as before, a little distorted; but there could not be more than about $\frac{3}{8}$ of the apparent diameter of the small star wanting to a complete occultation. Most probably, the path of the motion is not quite central; if so, the disk will remain a little distorted, during the whole time of the conjunction. Our present observations cannot determine which of the stars is at the greatest distance; but this will occasion no difference in the appearance; for, if the small star should be the nearest, its light will be equally lost in the brightness of the large one.

The observations I have made on this star, are not sufficient to direct us in the investigation of the nature of the motion by which this change is occasioned.

We may however be certain, that with regard to

Single Motions,

(a, b) Neither x nor ζ can be supposed to be the only moving bodies, without contradicting the highly probable arguments for the sun's motion.

(c, d) If we admit the sun to be the moving body, the stars ζ and x being at rest, we may calculate the effect of the solar parallax upon them, as follows. Let O move towards λ Herculis, with the annual velocity 1, as in the case of α Geminorum; then, from the situation and magnitude of the large star of ζ Herculis, which we will suppose 4m, the effect of the solar motion at ζ will be only ,0522; and, at x, supposed to be at the distance 5m,

it will be ,0418. This will show itself at the parallel of ζ in a direction of 25° 5′ north preceding, the solar motion being in the opposite direction south following. But this parallax will only produce, in 20 years and 10 months, an apparent change of 0″,444 in ζ, and of 0″,355 in x; and will separate the stars, instead of bringing them to a conjunction.

(*e*) A considerable advantage may be gained, by placing x at a little more than $\frac{1}{3}$ the distance of ζ from O. For as, in the abovementioned time, this would make the effect of parallax upon it 1″,18, a conjunction should now take place. But then the stars, though very near each other, would not be quite in contact; much less could one of them occasion an occultation of the other. The supposition also, that the small star should be only $\frac{1}{3}$ of the distance of the large one from us, is not very favourable to the hypothesis.

δ *Serpentis.*

This double star has undergone a very considerable change in the angle of position, but none in the distance of the two stars. The 5th of September, 1782, an accurate measure of the position was 42° 48′ south preceding; and February 7, 1802, it measured 61° 27′ south preceding. In 19 years and 155 days, therefore, the small star has moved, in a retrograde order, over an arch of 18° 39′.

Every argument, to examine the cause of this motion, which has been used with ε Bootis, in the paragraphs from (*a*) to (*i*), will completely apply to this star; from this we may conclude, that the most natural way of accounting for the observed changes, is to admit the two stars to form a binary system. In this case we calculate, with considerable probability, that the periodical

time of a revolution of the small star round δ Serpentis, must be about 375 years.

γ Virginis.

This double star, which has long been known to astronomers,* has undergone a visible change since the year 1780, when I first began my observations of it. The 21st of November, 1781, I measured the position of the two stars, which was 40° 44′ south following. The stars are so nearly equal, that I have but lately ascertained the following one to be rather larger than its companion; the position, therefore, ought now to be called north preceding. By a mean of three measures, that were taken on the 15th of April, 1803, the angle was 30° 20′ *np*.

The distance, as far as estimations by the diameter can determine, when the stars are so far asunder as these are, remains without alteration. May 21, 1781, they were $2\frac{1}{2}$ diameters asunder; and, by estimations lately made, with the same instrument and power as were used 21 years ago, the stars are still at the same distance of $2\frac{1}{2}$ diameters.

A very small proper motion in declination, of 0″,02 towards the south, has been assigned to this double star;† but the quantity is hardly sufficient for us to rely much upon the accuracy of the determination. I shall therefore rather consider γ Virginis as one of the stars of which we have no proper motion ascertained; and the arguments to which I shall refer, will consequently be those which have been given with ε Bootis.

The change of the angle of position, in the time of 21 years and 145 days, amounts to 10° 24′; from which we obtain the

* *Memoires de l'Academie des Sciences.* Ann. 1720.

† *Connoissance des Temps,* Année VI. page 213.

annual motion of 29′,16. The observed and calculated angles, with their differences, on which it will not be necessary to make any remarks, are in the following Table.

Times of the observations.	Observed angles.	Calculated angles.	Differences.
Nov. 21, 1781 - -	40° 44′	40° 44′	0° 0′
Jan. 29, 1802 - -	28 22	30 51	− 2 29
April 15, 1803 - -	30 20	30 20	0 0
May 28, 1803 - -	32 2	30 17	+ 1 45

As a confirmation of the accuracy of these observations, we may have recourse to a position of the same stars, deduced from the places of them, as they are given in MAYER's Zodiacal Catalogue. By two observations, reduced to the beginning of the year 1756, the preceding one was 3″,8 before the other in right ascension, and 5″,3 more north than that star. From this we calculate the position, which was 54° 21′ 37″ north preceding. The interval from the 1st of January, 1756, to the 21st of November, 1781, is 25 years and 325 days. When this is added to the period I have given, we have 47 years and 105 days, for a motion of 24° 2′. The annual motion, deduced from this lengthened period, which is 30′,5, differs less than 1½ minute from that which has been calculated from my observations. With the assistance, therefore, of MAYER's observation, which greatly supports our calculation, we may conclude, that the two stars of γ Virginis revolve round each other in about 708 years.

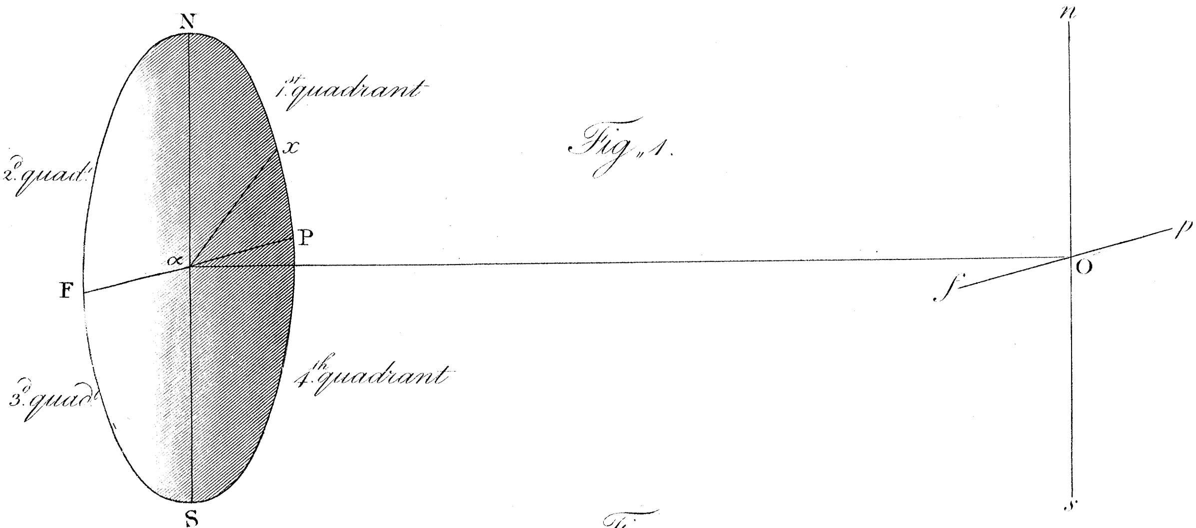

N
1st quadrant
x
2d quadt
P
α
F
4th quadrant
3d quadt
S
n
p
O
f
s

Fig. 1.

Fig. 2.

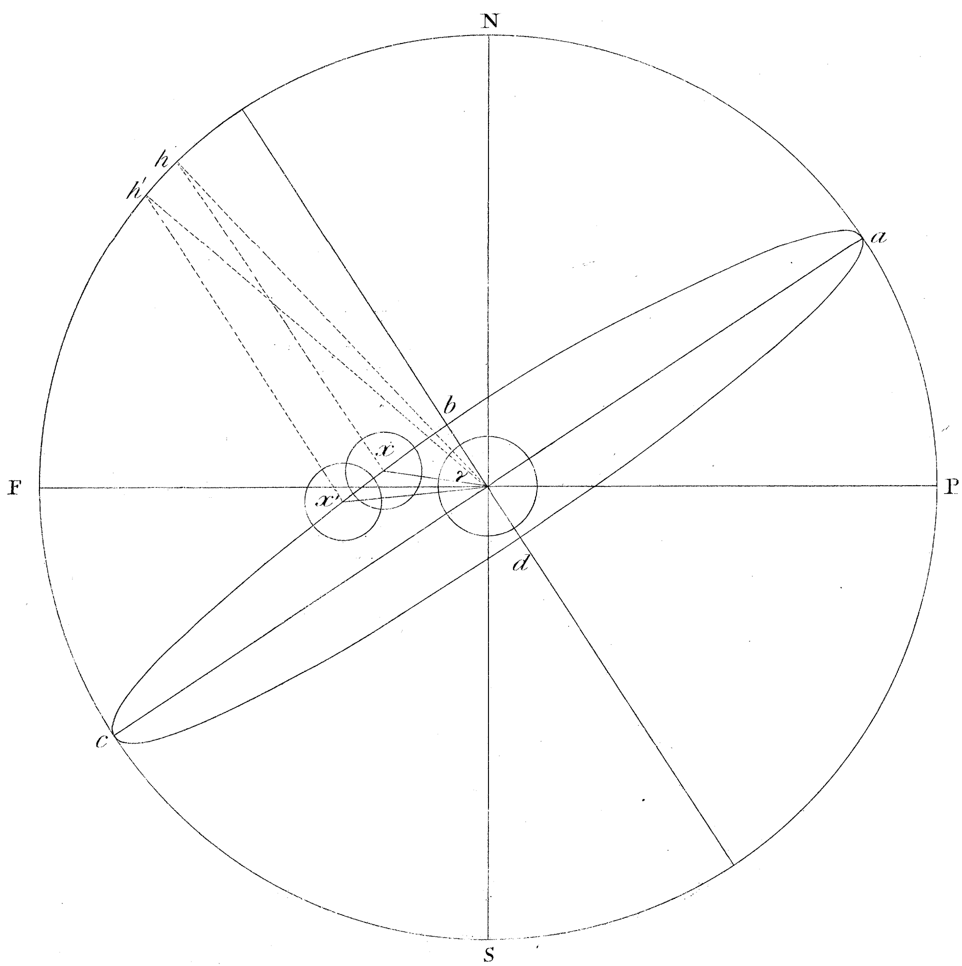
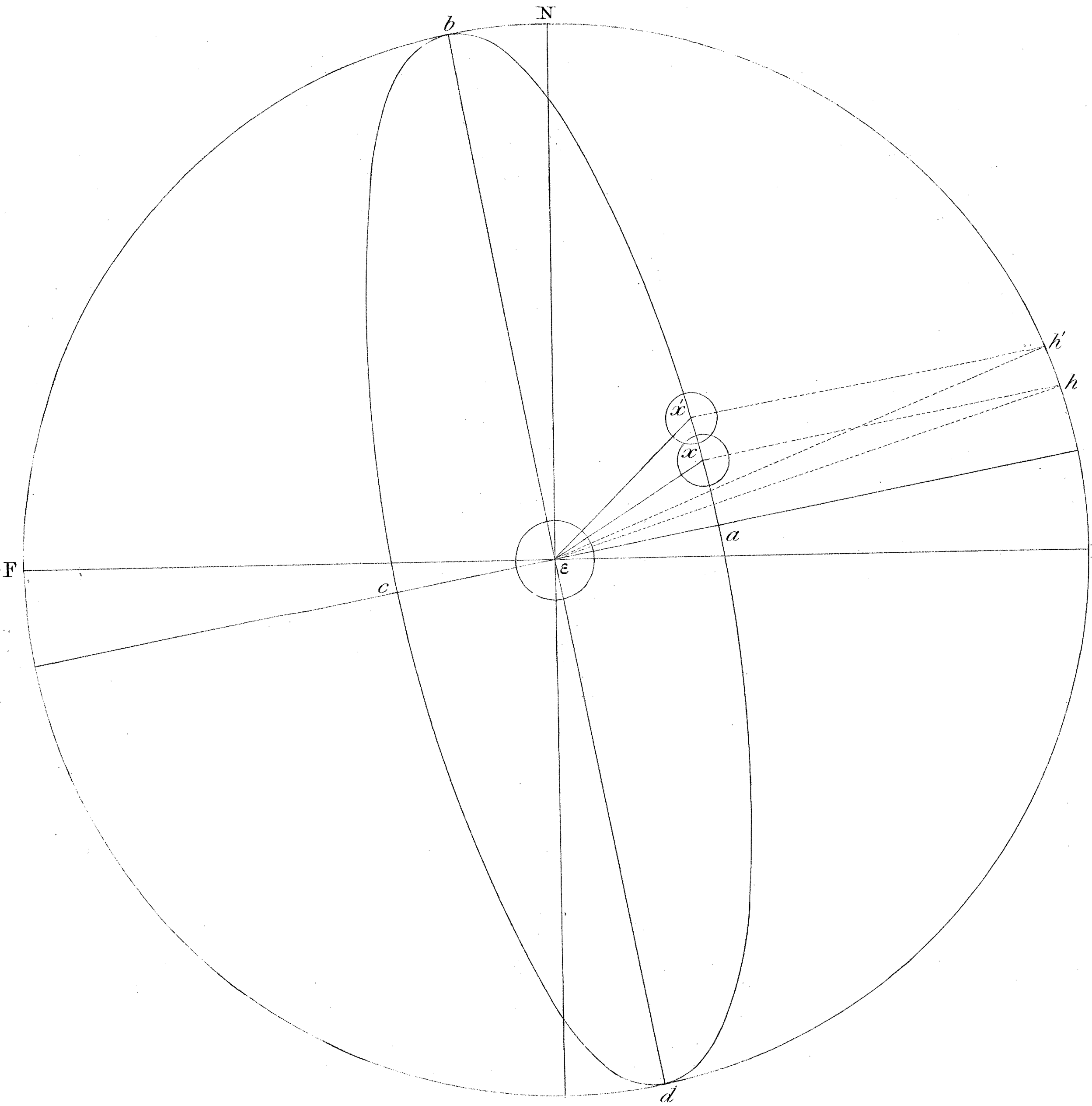

N
h
h'
a
b
x
F
x'
r
P
d
c
S

Fig. 3.



Basire sc.